3.7 #57 Solution

First find the slope of the normal lines

$$xy + 2x - y = 0$$

$$\frac{d}{dx}[xy + 2x - y = 0] \rightarrow x\frac{dy}{dx} + y(1) + 2 - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x-1) = -y-2 \qquad \frac{dy}{dx} = \frac{-y-2}{x-1} = \text{slope Tangent}$$

$$\text{Slope normal} = -\frac{dx}{dy} = \frac{x-1}{y+2} \quad \text{: parallel to } 2x+y=0$$

$$y = -2x$$

$$\frac{x-1}{y+2} = -2 \xrightarrow[\text{multiply}]{\text{Cross}} x-1 = -4-2y$$

$$\boxed{x = -3-2y}$$

* Sub this into original *

$$(-3-2y)y + 2(-3-2y) - y = 0$$

$$-3y - 2y^2 - 6 - 4y - y = 0$$

$$2y^2 + 8y + 6 = 0 \rightarrow y^2 + 4y + 3 = 0$$

$$(y+3)(y+1) = 0$$

$$y = -3 \text{ or } y = -1$$

Sub into original to find x vals

$y = -3$: $-3x + 2x + 3 = 0$

$x = 3$

$(3, -3)$: $\rightarrow$ equation $y + 3 = -2(x-3)$

$y = -1$: $-x + 2x + 1 = 0$.

$x = -1$

$(-1, -1)$: $\rightarrow$ eqn $y + 1 = -2(x+1)$

HW: Solutions Lesson 3

4.6

16.

Given $\frac{dV}{dt} = 10\ m^3/min$

Find $\frac{dh}{dt}$ when $h = 4$

$h = \frac{3}{8} b$

$h = \frac{3}{8}(2r)$

$h = \frac{3}{4} r$

$r = \frac{4h}{3}$

Rel $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$* \ V = \frac{16\pi}{27} h^3$

$\frac{dV}{dt} = \frac{16\pi}{9} h^2 \frac{dh}{dt}$

$10 = \frac{16\pi}{9}(16)\frac{dh}{dt}$

$\frac{90}{256\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{45}{128\pi}\ m/min = \frac{4500}{128\pi}\ cm/min$

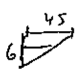
$\frac{dr}{dt} = \frac{4}{3}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{4}{3}\left(\frac{45}{128\pi}\right) = \frac{60}{128\pi} = \frac{15}{32\pi}\ m/min \checkmark = \frac{1500}{32\pi}\ cm/m = \frac{375}{8\pi}\ cm/min$

17.

Given $\frac{dV}{dt} = -50\ m^3/min$

$\frac{dh}{dt}$ when $h = 5\ m$



$\frac{6}{h} = \frac{45}{r}$

$6r = 45h$

$r = \frac{15}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$V = \frac{225\pi}{12} h^3$

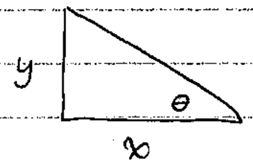
$\frac{dV}{dt} = \frac{225\pi}{4} h^2 \frac{dh}{dt}$

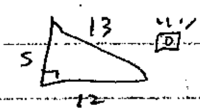
$-50 = \frac{225\pi}{4}(25)\frac{dh}{dt}$

$\frac{-8}{225\pi} = \frac{dh}{dt} = \frac{-800}{225\pi}\ cm/min = \frac{-32}{9\pi}\ cm/min$

$\frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt} = \frac{15}{2}\left(\frac{-8}{225\pi}\right) = \frac{-60}{225\pi}\ m/min = \frac{-80}{3\pi}\ cm/m$

19]





a] Given | $\frac{dx}{dt} = 5$ ft/sec

Find | $\frac{dy}{dt}$ when $x = 12$

Rel | $x^2 + y^2 = 13^2$

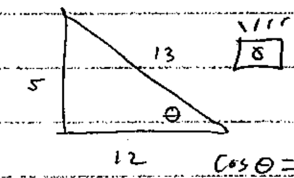
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\cancel{2}(12)(5) + \cancel{2}(5)\frac{dy}{dt} = 0 \qquad \frac{dy}{dt} = -12 \text{ ft/sec}$$

b] Find | $\frac{dA}{dt}$ when $y = 12$

Rel | $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + y\frac{1}{2}\frac{dx}{dt}$$



$$\frac{dA}{dt} = \frac{1}{2}(12)(-12) + \frac{5}{2}(5) = \frac{-144}{2} + \frac{25}{2} = \frac{-119}{2} \text{ ft}^2\text{/sec}$$

$\cos\theta = \frac{12}{13}$

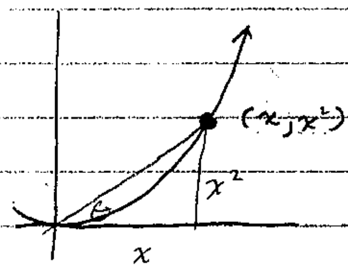
c] Rel | $\sin\theta = \frac{y}{13}$

$$\cos\theta\frac{d\theta}{dt} = \frac{1}{13}\frac{dy}{dt}$$

$$\frac{12}{13}\frac{d\theta}{dt} = \frac{1}{13}(-12)$$

$$\frac{d\theta}{dt} = -1 \text{ rad/sec}$$

25]

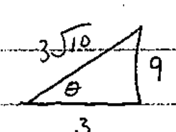


Given | $\frac{dx}{dt} = 10$ m/s Find | $\frac{d\theta}{dt}$ when $x = 3$

Rel | $\tan\theta = \frac{x^2}{x} = x$

9+81

$$\sec^2\theta\frac{d\theta}{dt} = \frac{dx}{dt}$$



$$10\frac{d\theta}{dt} = 10 \qquad \frac{d\theta}{dt} = 1 \text{ rad/sec}$$

$\cos\theta = \frac{1}{\sqrt{10}}$

$\sec\theta = \sqrt{10}$

1987#5

a) $V = \frac{1}{2} b \cdot h \cdot l = \frac{1}{2}(2)(3)(5) = 15 \text{ ft}^3$

b) Given| $\frac{dV}{dt} = -2$

Find| $\frac{dh}{dt}$ when $V = \frac{15}{4}$

RELATIONSHIP| $V = \frac{1}{2} b \cdot h \cdot 5 = \frac{5}{2} b \cdot h$

$\frac{b}{h} = \frac{2}{3} \rightarrow b = \frac{2h}{3}$

UPDATE| $V = \frac{5}{3} h^2$ $\qquad V = \frac{15}{4} \quad h^2 = \frac{9}{4} \quad h = \frac{3}{2}$

$\frac{d}{dt}\left[V = \frac{5}{3} h^2\right]$

$\frac{dV}{dt} = \frac{10}{3} h \frac{dh}{dt}$

$-2 = \frac{10}{3}\left(\frac{3}{2}\right)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-2}{5} \text{ ft/min}}$

c) Given| $\frac{dh}{dt} = \frac{-2}{5}$ (from part b)

Find| $\frac{dA}{dt}$ when $V = \frac{15}{4}$ or $h = \frac{3}{2}$

Relationship| $A = 5 \cdot b$

UPDATE| $A = \frac{10}{3} h$

$\frac{dA}{dt} = \frac{10}{3}\frac{dh}{dt} = \frac{10}{3}\left(\frac{-2}{5}\right) = \frac{-4}{3} \text{ ft}^2/\text{min}$

3.7 A.P QUESTION

a) $xy^2 - x^3y = 6$

$x(2y\frac{dy}{dx}) + y^2(1) - x^3(\frac{dy}{dx}) + y(-3x^2) = 0$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ $\quad 2xy - x^3 \neq 0$

b) $(1)y^2 - (1)^3y = 6$ $\quad y^2 - y = 6$ $\quad y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3,\ y = -2$

POINT	$\frac{dy}{dx}$	
$(1,3)$:	$\frac{3(3)-9}{6-1} = 0$	$\boxed{y = 3}$
$(1,-2)$:	$\frac{3(-2)-4}{-4-1} = \frac{-10}{-5} = 2$	$\boxed{y + 2 = 2(x-1)}$

c) vertical when $\frac{dy}{dx}$ DNE when $2xy - x^3 = 0$

$2xy = x^3$

$2y = x^2$

$y = x^2/2$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$-\frac{x^5}{4} = 6$

$-x^5 = 24$

$x = \sqrt[5]{-24}$