NOTES 4.6 #11

a) Given $\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$

Find $\frac{dr}{dt}$ when $r = 5$ ft

Rel $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

III D $100\pi = 4\pi(5)^2 \frac{dr}{dt}$

$\therefore \boxed{\frac{dr}{dt} = 1 \text{ ft/min}}$

b) Given $\frac{dr}{dt} = 1$ ft/min

Find $\frac{dA}{dt}$ when $r = 5$ ft

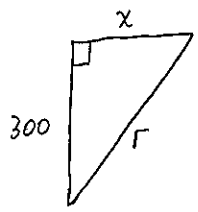
Rel $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

III D $\frac{dA}{dt} = 8\pi(5)(1) = 40\pi \text{ ft}^2/\text{min}$

$\boxed{\frac{dA}{dt} = 40\pi \text{ ft}^2/\text{min}}$

NOTES 4.6 #14

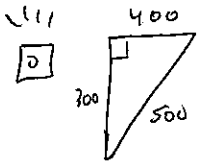


Given $\frac{dx}{dt} = 25$ ft/sec

Find $\frac{dr}{dt}$ when $r = 500$ ft

Rel ~~$300^2 + r^2 = x^2$~~ 300^2+x^2=r^2

III D



$$2r\frac{dr}{dt} = 2x\frac{dx}{dt}$$

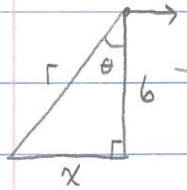
III D $2(500)\frac{dr}{dt} = 2(400)(25)$

$\boxed{\frac{dr}{dt} = 20 \text{ ft/sec}}$

HW SOLUTIONS

#21 Given $\frac{dr}{dt} = -2$ ft/sec

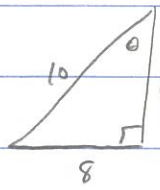
A] Find $\frac{dx}{dt}$ when $r = 10$



→ Relationship $x^2 + 36 = r^2$

$2x\frac{dx}{dt} = 2r\frac{dr}{dt}$

SNAP SHOT



$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\sec\theta = \frac{5}{3}$

$2(8)\frac{dx}{dt} = 2(10)(-2)$

$\frac{dx}{dt} = -\frac{5}{2}$ ft/sec

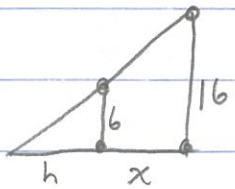
B] Find $\frac{d\theta}{dt}$ when $\frac{dx}{dt} = -\frac{5}{2}$ ft/sec

→ Relationship $\tan\theta = \frac{x}{6}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{6}\cdot\frac{dx}{dt}$

$\left(\frac{5}{3}\right)^2\frac{d\theta}{dt} = \frac{1}{6}\left(-\frac{5}{2}\right)$ $\quad \frac{d\theta}{dt} = -\frac{5}{12}\cdot\frac{9}{25} = -\frac{3}{20}$ radians/sec

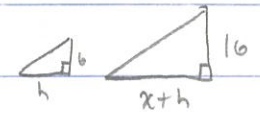
#29



Given $\frac{dx}{dt} = -5$ ft/sec

Find $\frac{dh}{dt}$

Relationship $\frac{16}{x+h} = \frac{6}{h}$ Similar triangles



$16h = 6x + 6h$

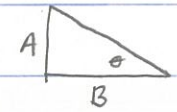
$16\frac{dh}{dt} = 6\frac{dx}{dt} + 6\frac{dh}{dt}$

$10\frac{dh}{dt} = 6\frac{dx}{dt}$

$10\left(\frac{dh}{dt}\right) = 6(-5)$

$\frac{dh}{dt} = -3$ ft/sec

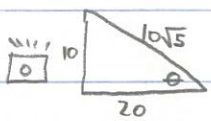
#34



Given $\frac{dA}{dt} = -2$ $\quad \frac{dB}{dt} = 1$

Find $\frac{d\theta}{dt}$ when $A = 10$ $\quad B = 20$

Rel $\tan\theta = \frac{A}{B}$ $\quad \sec^2\theta\frac{d\theta}{dt} = \frac{B\left(\frac{dA}{dt}\right) - A\left(\frac{dB}{dt}\right)}{B^2}$



$\left(\frac{\sqrt{5}}{2}\right)^2\frac{d\theta}{dt} = \frac{20(-2) - 10(1)}{(20)^2}$

$\cos\theta = \frac{20}{10\sqrt{5}} = \frac{2}{\sqrt{5}}$

$\sec\theta = \frac{\sqrt{5}}{2}$

$\frac{d\theta}{dt} = -\frac{1}{10}$ radians/sec

Convert after

$\frac{-1\text{ rad}}{10\text{ sec}}\cdot\frac{180\text{ deg}}{\pi\text{ rad}} = \frac{-18}{\pi}$ deg/sec

∴ I will ask for rad/sec not deg/sec

1990 AB4

Given $\frac{dr}{dt} = 0.04 = \frac{4}{100} = \frac{1}{25}$ cm/sec

A] Find $\frac{dV}{dt}$ when $r = 10$

Rel $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (10)^2 \left(\frac{1}{25}\right) = \underline{16\pi}$ cm^3/sec

B] Find $\frac{dA}{dt}$ when $V = 36\pi \rightarrow 36\pi = \frac{4}{3}\pi r^3$

$r^3 = 27 \rightarrow r = 3$

Rel $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi (3)\left(\frac{1}{25}\right) = \underline{\frac{6\pi}{25}}$ cm^2/sec

C] Find r when $\frac{dV}{dt} = \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dr}{dt}$

$4\pi r^2 \frac{dr}{dt} = \frac{dr}{dt}$

$4\pi r^2 = 1$

$r^2 = \frac{1}{4\pi}$

$r = \sqrt{\frac{1}{4\pi}}$ cm

DAY 2 HW 3.7

56 $x^2 + 2xy - 3y^2 = 0$

$2x + 2x\frac{dy}{dx} + y(2) - 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x - 6y) = -2y - 2x$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \left.\frac{-2y-2x}{2x-6y}\right|_{(1,1)} = \left.\frac{y+x}{3y-x}\right|_{(1,1)} = \frac{2}{2} = 1$

$m_T = 1 \quad m_N = -1$

$y - 1 = -(x-1) \longrightarrow y = -x + 2$

where do $x^2 + 2xy - 3y^2 = 0$ & $y = -x + 2$ intersect? (substitution / system of equations)

$x^2 + 2x(2-x) - 3(2-x)^2 = 0$

$x^2 + 4x - 2x^2 - 12 + 12x - 3x^2 = 0$

$-4x^2 + 16x - 12 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or $x = 3$

$x = 1$ → already known

$y = -(3) + 2 = -1$

$\boxed{(3, -1)}$