

3.7 HW Solutions Lesson 1 Solutions 3.7

1. $\frac{d}{dx} [x^2 y + x y^2 = 6]$

$$x^2 \frac{dy}{dx} + y(2x) + x(2y) \frac{dy}{dx} + y^2 = 0$$

$$\frac{dy}{dx} = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

5. $\frac{d}{dx} [x = \tan y]$

$$1 = \sec^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} \text{ or } \cos^2 y$$

29. $\frac{d}{dx} [y^2 = x^2 + 2x]$

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$2y \frac{dy}{dx} = 2x + 2$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left[\frac{x+1}{y} \right] = \frac{y(1) - (x+1) \frac{dy}{dx}}{y^2} = \frac{y - (x+1) \left(\frac{x+1}{y} \right)}{y^2}$$

49. $x^2 + xy + y^2 = 7$

crosses when $y = 0$

$$x^2 + x(0) + (0)^2 = 7$$

$$x^2 = 7$$

$$x = \pm\sqrt{7} \quad (\sqrt{7}, 0), (-\sqrt{7}, 0)$$

$$\frac{d}{dx} [x^2 + xy + y^2 = 7]$$

$$2x + x \frac{dy}{dx} + y(1) + 2y \frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} (x + 2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

$$\left. \frac{dy}{dx} \right|_{(\sqrt{7}, 0)} = \frac{-2\sqrt{7} - 0}{\sqrt{7} + 2(0)} = (-2)$$

$$\left. \frac{dy}{dx} \right|_{(-\sqrt{7}, 0)} = \frac{-2(-\sqrt{7}) - 0}{-\sqrt{7} + 2(0)} = (-2)$$

56.

$$3.7 \# 17 \quad x^2 + xy - y^2 = 1$$

$$\frac{d}{dx} [x^2 + xy - y^2 = 1]$$

$$2x + x \frac{dy}{dx} + y - 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (x - 2y) = -y - 2x$$

$$\frac{dy}{dx} = \frac{-y - 2x}{x - 2y}$$

$$\left. \frac{dy}{dx} \right|_{(2,3)} = \frac{-3-4}{2-6} = \frac{-7}{-4} = \frac{7}{4} \quad : \begin{array}{l} y-3 = \frac{7}{4}(x-2) \quad T \\ y-3 = -\frac{4}{7}(x-2) \quad N \end{array}$$

$$3.7 \# 21 \quad [6x^2 + 3xy + 2y^2 + 17y - 6 = 0]'$$

$$12x + 3xy' + 3y + 4yy' + 17y' = 0$$

$$y'(3x + 4y + 17) = -12x - 3y$$

$$y' = \frac{-12x - 3y}{3x + 4y + 17}$$

$$y'(-1, 0) = \frac{12}{-3+17} = \frac{12}{14} = \frac{6}{7} \quad : \begin{array}{l} y = \frac{6}{7}(x+1) \quad T \\ y = -\frac{7}{6}(x+1) \quad N \end{array}$$