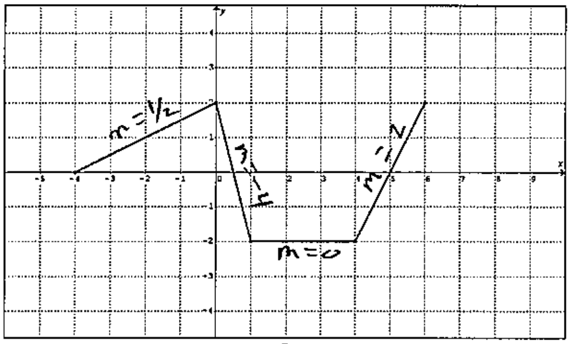
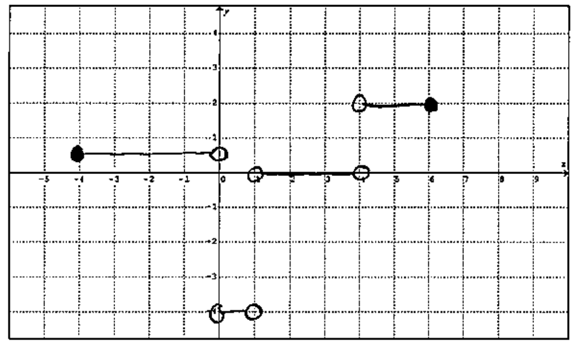
LESSON 3 HW: Solutions

~~LESSON 2 – HW~~:

1. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end. Graph $y = f'(x)$ and state its domain.



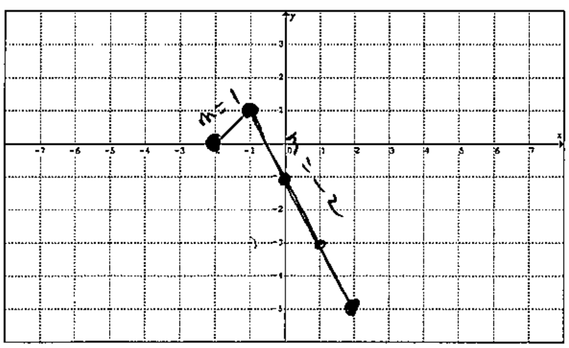
$y = f(x)$



$y = f'(x)$

2. Sketch the graph of a continuous function with domain [-2,2], $f(0) = -1$, and

$$f'(x) = \begin{cases} 1; & x < -1 \\ -2; & x > -1 \end{cases}.$$



$y = f(x)$

3. Using the information from problem 2, write an equation of the line tangent to f at $x = 0$.

$f(0) = -1$

$f'(0) = -2$

$y + 1 = -2(x)$

(4.) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{5(x+h)^2 - 2(x+h) + 1 - [5x^2 - 2x + 1]}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{5x^2} + 10xh + 5h^2 \cancel{- 2x} - 2h \cancel{+ 1} \cancel{- 5x^2} \cancel{+ 2x} \cancel{- 1}}{h}$

$= \lim_{h \to 0} \dfrac{h(10x + 5h - 2)}{h} = \lim_{h \to 0} 10x + 5h - 2 = 10x - 2$

$\boxed{f'(x) = 10x - 2}$

(5.) $f'(x) = 0$

$10x - 2 = 0$

$\boxed{x = 1/5}$

(6.) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h \to 0} \dfrac{\frac{x+1 - (x+h+1)}{(x+1)(x+h+1)}}{h}$

$= \lim_{h \to 0} \dfrac{\frac{-h}{(x+1)(x+h+1)}}{h} = \lim_{h \to 0} \dfrac{-1}{(x+1)(x+h+1)} = \dfrac{-1}{(x+1)^2}$

(7) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$

$= \lim_{h \to 0} \dfrac{\left(\sqrt{2(x+h)+1} - \sqrt{2x+1}\right)}{h} \dfrac{\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}{\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$

$= \lim_{h \to 0} \dfrac{2(x+h) + 1 - (2x+1)}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} = \lim_{h \to 0} \dfrac{2h}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$

$= \lim_{h \to 0} \dfrac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} = \dfrac{2}{2\sqrt{2x+1}} = \dfrac{1}{\sqrt{2x+1}}$

8 $f(x) = \begin{cases} x+1 \ ; & x>1 \\ -x^2+3x \ ; & x \le 1 \end{cases}$

when $x>1$: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)+1-(x+1)}{h}$

$= \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1$

when $x<1$: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-(x+h)^2+3(x+h)-(-x^2+3x)}{h}$

$= \lim_{h\to 0} \frac{-x^2-2xh-h^2+3x+3h+x^2-3x}{h}$

$= \lim_{h\to 0} -2x-h+3 = -2x+3$

when $x=1$: $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = ?$

From HW 2

$$\left.\begin{array}{l} f'_+(1) = \cdots = 1 \\ f'_-(1) = \cdots = 1 \end{array}\right\} \therefore f'(1) = 1$$

so...

$$f'(x) = \begin{cases} 1 \ ; & x \ge 1 \\ -2x+3 \ ; & x<1 \end{cases}$$

Please note we could have put the equals with either part because the function has same derivative from right or left

could have said $f'(x) = \begin{cases} 1 \ ; & x>1 \\ -2x+3 \ ; & x \le 1 \end{cases}$

(9) $m'(1) = \lim_{h\to 0} \dfrac{m(1+h) - m(1)}{h} = ?$

$$m'_+(1) = \lim_{h\to 0^+} \frac{m(1+h) - m(1)}{h} = \lim_{h\to 0^+} \frac{\sqrt{1+h} - [1]}{h}$$

$$= \lim_{h\to 0^+} \frac{(\sqrt{1+h} - 1)}{(h)} \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h} + 1)}$$

$$= \lim_{h\to 0^+} \frac{1+h-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0^+} \frac{1}{\sqrt{1+h}+1} = \frac{1}{2}$$

$$m'_-(1) = \lim_{h\to 0^-} \frac{m(1+h) - m(1)}{h} = \lim_{h\to 0^-} \frac{\frac{1+h}{2} + \frac{1}{2} - [1]}{h}$$

$$= \lim_{h\to 0^-} \frac{\frac{1}{2} + \frac{h}{2} + \frac{1}{2} - 1}{h} = \lim_{h\to 0^-} \frac{\frac{h}{2}}{h}$$

$$= \lim_{h\to 0^-} \frac{1}{2} = \frac{1}{2}$$

$\therefore \lim_{h\to 0} \dfrac{m(1+h) - m(1)}{h} = \dfrac{1}{2} \quad m'(1) = \dfrac{1}{2}$

m is differentiable at $x = 1$

m is <u>smooth</u> at $x = 1$