# AB.Q103.L2 HW SOLUTIONS

① a) $$\text{ave rate }\Delta_{[-1,2]} = \frac{f(2)-f(-1)}{2-(-1)} = \frac{3-(-4)}{3} = \boxed{\frac{7}{3}}$$

b) $$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = ?$$

□ $$f'_+(1) = \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{(1+h)+1-[2]}{h}$$

$$= \lim_{h\to 0^+} \frac{h}{h} = \lim_{h\to 0^+} 1 = 1$$

□ $$f'_-(1) = \lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{-(1+h)^2+3(1+h)-[2]}{h}$$

$$= \lim_{h\to 0^-} \frac{-1-2h-h^2+3+3h-2}{h} = \lim_{h\to 0^-} -h+1 = 1$$

$$\therefore \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = 1 \text{ ; i.e } \boxed{f'(1) = 1}$$

The graph of $f$ is smooth at $x = 1$

c) $f(1)=2$ $f'(1)=1$ $\rightarrow$ $\boxed{y-2=(x-1)}$

② a) i) $b(1) = -2+1 = -1$

ii) $\lim_{x\to 1^+} b(x) = \lim_{x\to 1^+} x = 1$

$\lim_{x\to 1^-} b(x) = \lim_{x\to 1^-} -2+x = -1$

$\therefore \lim_{x\to 1} b(x)$ DNE

iii) $\lim_{x\to 1} b(x) \neq b(1)$

$\therefore$ b is not continuous at $x=1$

⇨

② b) $b'(1) = \lim_{h\to 0} \frac{b(1+h) - b(1)}{h} = ?$

□ $b'_+(1) = \lim_{h\to 0^+} \frac{b(1+h) - b(1)}{h} = \lim_{h\to 0^+} \frac{(1+h) - (-2+1)}{h}$

$= \lim_{h\to 0^+} \frac{2+h}{h} = \lim_{h\to 0^+} \frac{2}{h} + \lim_{h\to 0^+} \frac{h}{h}$

$= \lim_{h\to 0^+} \frac{2}{h}^{\nearrow \infty} + \lim_{h\to 0^+} 1 \longrightarrow DNE$

OPTIONAL: □ $b'_-(1) = \lim_{h\to 0^-} \frac{b(1+h) - b(1)}{h} = \lim_{h\to 0^-} \frac{-2+(1+h) - (-2+1)}{h}$

$= \lim_{h\to 0^-} \frac{h}{h} = \lim_{h\to 0^-} 1 = 1$

$\therefore \lim_{h\to 0} \frac{b(1+h) - b(1)}{h}$ DNE ; $b'(1)$ DNE

The function b is not differentiable at $x=1$
(DISCONTINUOUS AT $x=1$)

c) If we were not required to use a definition we could have simply stated.

b is not continuous at $x=1$

$\therefore$ b is not differentiable at $x=1$ (THM)

(3) $f'(1.57) \approx \dfrac{f(1.74) - f(1.39)}{1.74 - 1.39} = \dfrac{1126 - 1255}{0.35}$

$= -368.57$

$f'(3) \approx \dfrac{f(3.24) - f(2.64)}{3.24 - 2.64} = \dfrac{805 - 869}{0.6}$

$= -106\frac{2}{3}$

(4) $f'(1) = \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = ?$

I $f'_+(1) = \lim_{h \to 0^+} \dfrac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \dfrac{\frac{1}{1+h} - [1]}{h} = \lim_{h \to 0^+} \dfrac{\frac{1}{1+h} - \frac{1+h}{1+h}}{h}$

$= \lim_{h \to 0^+} \dfrac{\frac{-h}{1+h}}{h} = \lim_{h \to 0^+} \dfrac{-1}{1+h} = -1$

II $f'_-(1) = \lim_{h \to 0^-} \dfrac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \dfrac{1+h - [1]}{h}$

$= \lim_{h \to 0^-} 1 = 1$

$\therefore \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$ DNE ; $f'(1)$ DNE

$f$ is not differentiable at $x = 1$

$f$ has a corner at $x = 1$

(5) i) $g(0) = 1 + e^0 = 2$

ii) $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} 2 + \sqrt{x} = 2$

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} 1 + e^{-x} = 2$

$\therefore \lim_{x \to 0} g(x) = 2$

iii) $\lim_{x \to 0} g(x) = g(0)$

$\therefore g$ is continuous at $x = 0$