AB. Q103 L1 HW SOLUTIONS

① (a) $y - 3 = 5(x-2)$

(b) $y - 3 = \frac{-1}{5}(x-2)$

Same point $(3,2)$

$m_{Normal} = -\text{reciprocal}$

② (a) $\underset{[-2,4]}{\text{Ave rate}\,\Delta} = \frac{f(4)-f(-2)}{4-(-2)} = \frac{(0)-(12)}{6} = \boxed{-2}$

(b) $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{[(1+h)^2-4(1+h)]-[-3]}{h}$

$= \lim_{h\to 0} \frac{1+2h+h^2-4-4h+3}{h} = \lim_{h\to 0} \frac{h(h-2)}{h} = \lim_{h\to 0} h-2 = \boxed{-2}$

(c) $f(1) = -3 \quad f'(1) = -2 \rightarrow \boxed{y+3 = -2(x-1)}$

③ (a) $\underset{[-2,4]}{\text{Ave rate}\,\Delta} = \frac{f(4)-f(-2)}{4-(-2)} = \frac{[12]-[2]}{6} = \boxed{\frac{5}{3}}$

(b) $f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = ?$

□ $f'_+(0) = \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{[(0+h)^2-(0+h)]-[0]}{h} = \lim_{h\to 0^+} \frac{h^2-h}{h} = \lim_{h\to 0^+} h-1 = -1$

□ $f'_-(0) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{-(0+h)-0}{h} = \lim_{h\to 0^-} \frac{-h}{h} = \lim_{h\to 0^-} -1 = -1$

$\therefore \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = -1$; i.e $f'(0) = -1$

The graph of $f(x)$ is smooth at $x = 0$

(c) $f(0) = 0 \quad f'(0) = -1$; $\boxed{y = -(x)}$

4 a) $\underset{[-4,0]}{\text{average rate}\,\Delta} = \dfrac{f(0)-f(-4)}{0-(-4)} = \dfrac{(0)-(-8)}{4} = \boxed{2}$

b) $f'(-2) = \lim\limits_{h\to 0} \dfrac{f(-2+h)-f(-2)}{h} = \,?$

$\square\; f'_{+}(-2) = \lim\limits_{h\to 0^+} \dfrac{f(-2+h)-f(-2)}{h} = \lim\limits_{h\to 0^+} \dfrac{-(-2+h)^2-[-4]}{h}$

$= \lim\limits_{h\to 0^+} \dfrac{-4+4h-h^2+4}{h} = \lim\limits_{h\to 0^+} \dfrac{h(4-h)}{h}$

$= \lim\limits_{h\to 0^+} 4-h = \underline{4}$

$\square\; f'_{-}(-2) = \lim\limits_{h\to 0^-} \dfrac{f(-2+h)-f(-2)}{h} = \lim\limits_{h\to 0^-} \dfrac{2(-2+h)-[-4]}{h}$

$= \lim\limits_{h\to 0^-} \dfrac{-4+2h+4}{h} = \lim\limits_{h\to 0^-} \dfrac{2h}{h} = \lim\limits_{h\to 0^-} 2 = \underline{2}$

$\therefore \lim\limits_{h\to 0} \dfrac{f(-2+h)-f(-2)}{h}$ DNE $\quad f'(-2)$ DNE

There is a corner at $x=-2$

c) The graph of f does not have a tangent at $x=-2$

⑤ a) $g'(2) = \lim_{h\to 0} \frac{g(2+h)-g(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

$= \lim_{h\to 0} \frac{\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{2(2+h)}}{h}$

Common denominator

h

$= \lim_{h\to 0} \frac{-1}{2(2+h)} = \boxed{\frac{-1}{4}}$

b) $g'(2) = \lim_{x\to 2} \frac{g(x)-g(2)}{x-2} = \lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$

$= \lim_{x\to 2} \frac{\frac{2}{2x} - \frac{x}{2x}}{x-2} = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2}$

$= \lim_{x\to 2} \frac{-(x-2)}{2x} \cdot \frac{1}{(x-2)} = \lim_{x\to 2} \frac{-1}{2x} = \boxed{-\frac{1}{4}}$

⑥

i) $P(-1) = \sec(-\pi) - \ln(1) = -1 - 0 = -1$

ii) $\lim_{x\to -1} P(x) = \lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1} \frac{(x+1)(x-1)}{x+1}$

$= \lim_{x\to -1} x-1 = -1-1 = -2$

iii) $\lim_{x\to -1} P(x) \neq P(-1)$

$\therefore$ p is not continuous at $x=-1$