AB Q102 L2 Hw Solutions

[redacted]

1. (i) $f(1) = (1)^2 - 2(1) + 3 = 2$

   (ii) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} x^2 - 2x + 3 = 2$

   (iii) $\lim\limits_{x\to 1} f(x) = f(1)$

   $\therefore$ f is continuous at $x = 1$

2. (i) $g(0) = 1 + e^0 = 2$

   (ii) $\lim\limits_{x\to 0^+} g(x) = \lim\limits_{x\to 0^+} 2 + \sqrt{x} = 2$

   $\lim\limits_{x\to 0^-} g(x) = \lim\limits_{x\to 0^-} 1 + e^{-x} = 2$

   $\therefore \lim\limits_{x\to 0} g(x) = 2$

   (iii) $g(0) = \lim\limits_{x\to 0} g(x)$

   $\therefore$ g is continuous at $x = 0$

3. (i) $r(-1) = 5$

   (ii) $\lim\limits_{x\to -1} r(x) = \lim\limits_{x\to -1} x + 2 = 1$

   (iii) $\lim\limits_{x\to -1} r(x) \neq r(-1)$

   $\therefore$ r is not continuous at $x = -1$

4. (i) $f(0) = 0^2 + 2 = 2$

   (ii) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2 + 2 = 2$

   $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{1}{x} \to -\infty$

   $\therefore \lim\limits_{x\to 0^+} f(x) \neq \lim\limits_{x\to 0^-} f(x)$

   $\therefore \lim\limits_{x\to 0} f(x)$ DNE

   (iii) $\lim\limits_{x\to 0} f(x) \neq f(0)$

   $\therefore$ f is not continuous at $x = 0$

5. (i) $g(3)$ DNE

   (iii) $\lim\limits_{x\to 3} g(x) \neq g(3)$

   $\therefore$ g is not continuous at $x = 3$

   B. $g(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$

6. (i) $d(4)$ DNE

   (iii) $\lim\limits_{x\to 4} d(x) \neq d(4)$

   $\therefore$ d is not cont. at $x = 4$

   B. $d(x) = \begin{cases} \frac{x-4}{\sqrt{x}-2} & x \neq 4 \\ 4 & x = 4 \end{cases}$

   $\lim\limits_{x\to 4} \frac{(x-4)}{\sqrt{x}-2} \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \lim\limits_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$

   $= \lim\limits_{x\to 4} \sqrt{x} + 2 = 4$

7. (i) $h(0) = 0$

   (ii) $-1 \leq \sin\frac{1}{x} \leq 1$

   $-x \leq x\sin\frac{1}{x} \leq x$

   $\lim\limits_{x\to 0} -x = 0 = \lim\limits_{x\to 0} x$

   $\therefore \lim\limits_{x\to 0} x\sin\frac{1}{x} = 0$ (Sandwich Thm)

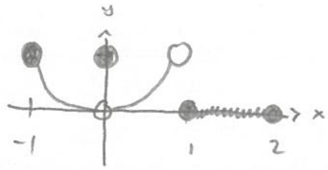
   (iii) $\lim\limits_{x\to 0} h(x) = h(0)$

   $\therefore$ h is continuous at $x = 0$

A.B. Q102 L2

HW: solutions

2.3 #23

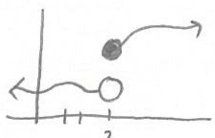


a] on $[-1,2]$ f is not continuous at $x=0,1$

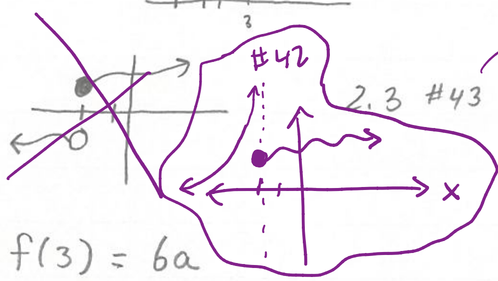
b] Removable disco at $x=0$ b/c $\lim_{x\to 0} f(x)$ exist and $f(0)$ exist, but $\lim_{x\to 0} f(x) \neq f(0)$

Jump disco at $x=1$ b/c $\lim_{x\to 1^+} f(x)$ exist and $\lim_{x\to 1^-} f(x)$ exist, but $\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$

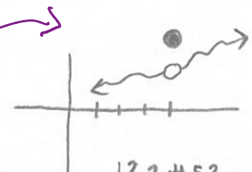
2.3 #41 ex:



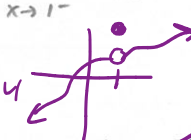
2.3 #42.



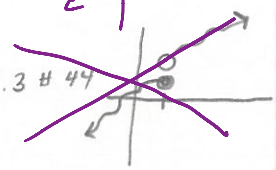
2.3 #43



#44



2.3 #44



2.2 #53

a) $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{1}{x} = 0$

b) $\lim_{x\to \infty} f(x) = \lim_{x\to\infty} -1 = -1$

c) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1}{x} = -\infty$

d) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -1 = -1$

2.3 #47 $f(3) = 6a$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x^2 - 1 = 8$

so $6a = 8$ $a = \frac{4}{3}$

2.2 #6 $f(x) = \frac{2x-1}{|x|-3} = \begin{cases} \frac{2x-1}{x-3} & x \geq 0 \\ \frac{2x-1}{-x-3} & x < 0 \end{cases}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x-1}{x-3} = 2$ graph approaches $y=2$ on far right end

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{2x-1}{-x-3} = -2$ graph approaches $y=-2$ on far left end

2.2 #9 $-1 \leq \cos x \leq 1$

$-1 \leq -\cos x \leq 1$

$0 \leq 1-\cos x \leq 2$

$0 \leq \frac{1-\cos x}{x^2} \leq \frac{2}{x^2}$

$\lim_{x\to\infty} 0 = 0$ $\therefore \lim_{x\to\infty} \frac{1-\cos x}{x^2} = 0$

$\lim_{x\to\infty} \frac{2}{x^2} = 0$

2.2 #22 $y = \left(\frac{2}{x}+1\right)\left(5-\frac{1}{x^2}\right)$

$\lim_{x\to -\infty} \left(\cancel{\frac{2}{x}}^{\to 0}+1\right)\left(5-\cancel{\frac{1}{x^2}}^{\to 0}\right) = 5$

$\lim_{x\to \infty} \left(\cancel{\frac{2}{x}}^{\to 0}+1\right)\left(5-\cancel{\frac{1}{x^2}}^{\to 0}\right) = 5$

2.2 #41: $\lim_{x\to\pm\infty} \frac{(x-2)}{(2x^2+3x-5)} \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}^{\to 0} - \cancel{\frac{2}{x^2}}^{\to 0}}{2+\cancel{\frac{3}{x}}^{\to 0}-\cancel{\frac{5}{x^2}}^{\to 0}} = \frac{0}{2} = 0$

H.A: $y=0$

2.2 #42: $\lim_{x\to\pm\infty} \frac{(3x^2-x+5)\left(\frac{1}{x^2}\right)}{(x^2-4)\left(\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{3-\cancel{\frac{1}{x}}^{\to 0}+\cancel{\frac{5}{x^2}}^{\to 0}}{1-\cancel{\frac{4}{x^2}}^{\to 0}} = 3$

H.A: $y=3$