Calculus AB: Q501 (POST AP) Section 6.3 Notes: Integration by Parts

Up to this stage we have been unable to evaluate integrals such as the following:

$\int \ln x dx, \quad \int xe^x dx, \quad \int x^2 \sin x dx, \quad \int \tan^{-1} x dx$

The next formula will enable us to evaluate not only these, but also many other types of integrals.

If $u = f(x)$ and $v = g(x)$ and if f' and g' are continuous, then $\int udv = uv - \int vdu$.

Proof:

Example 1: Evaluate $\int xe^{2x} dx$.

Example 2: Evaluate $\int x \sec^2 x dx$

Example 3: Evaluate $\int \ln x dx$

Example 4: Evaluate $\int x^2 e^{2x} dx$.

Example 5: Evaluate $\int e^x \cos x dx$.

Evaluate $\int \sec^3 x dx$.

Tabular integration

(Chapter 6.3) **Integration by Parts** Supplement Homework/Practice

Evaluate each integral.

1. $\int xe^{-x}dx$

2. $\int x\sec x\tan xdx$

3. $\int \tan^{-1} xdx$

4. $\int e^{-x}\sin xdx$

5. $\int \sin x\ln(\cos x)dx$

6. $\int \csc^3 xdx$

7. $\int e^{4x}\sin 5xdx$

8. $\int \cos\sqrt{x}dx$

AB – Additional Integration Problems Involving “Integration by Parts”

Evaluate each integral using the method of “integration by parts”.

You may wish to use tabular integration by parts if applicable

1. $\int x^2 e^{-x} dx$

2. $\int x^2 \sqrt{x-1} dx$

3. $\int x \sec^2 x dx$

4. $\int \sin^{-1} x dx$

5. $\int \sqrt{x} \ln x dx$

Calculus AB: Q501 (POST AP) Section 6.5 Notes: Integration by Partial Fractions – LESSON 2

Integration with Partial Fractions Practice

TEXT: Pg. 369 5 – 13 odd (The pdf for this packet includes these problems)

Quick Review 6.5 *(For help, go to Sections 2.2 and 2.3.)*

In Exercises 1–4, use the polynomial division algorithm (as in Example 2 of this section) to write the rational function in the form $Q(x) + \dfrac{R(x)}{D(x)}$, where the degree of R is less than the degree of D.

1. $\dfrac{x^2}{x-1}$
2. $\dfrac{x^2}{x^2-4}$
3. $\dfrac{x^2+x+1}{x^2+x-2}$
4. $\dfrac{x^3-5}{x^2-1}$

In Exercises 5–10, let $f(x) = \dfrac{60}{1+5e^{-0.1x}}$.

5. Find where f is continuous.
6. Find $\lim\limits_{x\to\infty} f(x)$.
7. Find $\lim\limits_{x\to-\infty} f(x)$.
8. Find the y-intercept of the graph of f.
9. Find all horizontal asymptotes of the graph of f.
10. Draw the graph of $y = f(x)$.

Section 6.5 Exercises

In Exercises 1–4, find the values of A and B that complete the partial fraction decomposition.

1. $\dfrac{x-12}{x^2-4x} = \dfrac{A}{x} + \dfrac{B}{x-4}$
2. $\dfrac{2x+16}{x^2+x-6} = \dfrac{A}{x+3} + \dfrac{B}{x-2}$
3. $\dfrac{16-x}{x^2+3x-10} = \dfrac{A}{x-2} + \dfrac{B}{x+5}$
4. $\dfrac{3}{x^2-9} = \dfrac{A}{x-3} + \dfrac{B}{x+3}$

In Exercises 5–14, evaluate the integral.

5. $\displaystyle\int \frac{x-12}{x^2-4x}\,dx$
6. $\displaystyle\int \frac{2x+16}{x^2+x-6}\,dx$
7. $\displaystyle\int \frac{2x^3}{x^2-4}\,dx$
8. $\displaystyle\int \frac{x^2-6}{x^2-9}\,dx$
9. $\displaystyle\int \frac{2\,dx}{x^2+1}$
10. $\displaystyle\int \frac{3\,dx}{x^2+9}$
11. $\displaystyle\int \frac{7\,dx}{2x^2-5x-3}$
12. $\displaystyle\int \frac{1-3x}{3x^2-5x+2}\,dx$
13. $\displaystyle\int \frac{8x-7}{2x^2-x-3}\,dx$
14. $\displaystyle\int \frac{5x+14}{x^2+7x}\,dx$

In Exercises 15–18, solve the differential equation.

15. $\dfrac{dy}{dx} = \dfrac{2x-6}{x^2-2x}$
16. $\dfrac{du}{dx} = \dfrac{2}{x^2-1}$
17. $F'(x) = \dfrac{2}{x^3-x}$
18. $G'(t) = \dfrac{2t^3}{t^3-t}$

In Exercises 19–22, find the integral *without* using the technique of partial fractions.

19. $\displaystyle\int \frac{2x}{x^2-4}\,dx$
20. $\displaystyle\int \frac{4x-3}{2x^2-3x+1}\,dx$
21. $\displaystyle\int \frac{x^2+x-1}{x^2-x}\,dx$
22. $\displaystyle\int \frac{2x^3}{x^2-1}\,dx$

In Exercises 23–26, the logistic equation describes the growth of a population P, where t is measured in years. In each case, find **(a)** the carrying capacity of the population, **(b)** the size of the population when it is growing the fastest, and **(c)** the rate at which the population is growing when it is growing the fastest.

23. $\dfrac{dP}{dt} = 0.006P(200-P)$
24. $\dfrac{dP}{dt} = 0.0008P(700-P)$
25. $\dfrac{dP}{dt} = 0.0002P(1200-P)$
26. $\dfrac{dP}{dt} = 10^{-5}P(5000-P)$

In Exercises 27–30, solve the initial value problem using partial fractions. Use a graphing utility to generate a slope field for the differential equation and verify that the solution conforms to the slope field.

27. $\dfrac{dP}{dt} = 0.006P(200-P)$ and $P = 8$ when $t = 0$.
28. $\dfrac{dP}{dt} = 0.0008P(700-P)$ and $P = 10$ when $t = 0$.
29. $\dfrac{dP}{dt} = 0.0002P(1200-P)$ and $P = 20$ when $t = 0$.
30. $\dfrac{dP}{dt} = 10^{-5}P(5000-P)$ and $P = 50$ when $t = 0$.

In Exercises 31 and 32, a population function is given.

(a) Show that the function is a solution of a logistic differential equation. Identify k and the carrying capacity.

(b) **Writing to Learn** Estimate $P(0)$. Explain its meaning in the context of the problem.

31. ***Rabbit Population*** A population of rabbits is given by the formula

$$P(t) = \frac{1000}{1+e^{4.8-0.7t}},$$

where t is the number of months after a few rabbits are released.

32. ***Spread of Measles*** The number of students infected by measles in a certain school is given by the formula

$$P(t) = \frac{200}{1+e^{5.3-t}},$$

where t is the number of days after students are first exposed to an infected student.

Calculus AB: Q501 (POST AP) Supplemental Notes: Integration by Trigonometric Substitutions –LESSON3

Expression in Integrand	Trigonometric Substitution
$a^2 - x^2$	$x = a\sin\theta$
$a^2 + x^2$	$x = a\tan\theta$
$x^2 - a^2$	$x = a\sec\theta$

Example 1: Evaluate $\int \frac{dx}{x^2\sqrt{16 - x^2}}$

Example 2: Evaluate $\int \frac{dx}{\sqrt{4+x^2}}$

Example 3: Evaluate $\int \frac{\sqrt{x^2-9}dx}{x}$

Lesson 3: Trigonometric Substitution Integration Practice

#42. $\int \frac{8dx}{x^2\sqrt{4-x^2}}$

#35. $\int \frac{dx}{\sqrt{9+x^2}}$

#37. $\int \frac{dx}{\sqrt{4x^2-49}}$

MISC EXTRA: $\int \sin^3 x\, dx$

Hint: $\int \sin^3 x\, dx = \int \sin^2 x \cdot \sin x\, dx = \int \left(1-\cos^2 x\right)\cdot \sin x\, dx$

… finish with a u-substitution $u = \cos x$

#39. $\int \frac{x^3}{\sqrt{1-x^2}}\, dx$

AB.Q501.REVIEW PRACTICE

1. $\int x^2 \ln x \, dx$

2. $\int x^2 e^{-2x} \, dx$

3. $\int \frac{11x+2}{2x^2-5x-3} \, dx$

4. $\int \frac{dx}{x^2\sqrt{25-x^2}}$

5. $\int \frac{dx}{1-x^2}$ A) Do it with partial Fractions B) Do it with a trigonometric substitution